Land soil erosion. CAESAR Numerical Simulation

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1 Introductory Comments

The original CAESAR model [2, 3] includes an erosion function, but it is likely to be proper to the erosion process on the bed river rather than on the hill slope. Both formula used in this model to calculate the sediment flux were established for channel or river flows [4, 15]. The mechanism of the erosion process on a strem bed is considerable different than the one on a hill slope and extrapolating the data from one of this cases to the other one often leads to erroneous results [9].

Also, the interactions among vegetation - water flow - soil erosion have a heuristic character rather than a physical basis. Our goal is to introduce a new module (based on physical principles) dedicated to overland erosion **in the presence of vegetation** that will work with the existing hydrodynamic modules in CAESAR. For our purposes, let us start from a theoretical model.

2 Basic mathematical model

To take into account the contribution of the vegetation to the water and erosion dynamics in a watershed, we begin with the model described by the following equations, [6, 14]

$$\partial_t \theta h + \partial_i(\theta h u^i) = \mathcal{W}, \tag{1a}$$

$$\partial_t(\theta h u^i) + \partial_j(\theta h u^i u^j) + \theta h g \delta^{ij} \partial_j(z + h) = \mathcal{H}^i, \quad i = \overline{1,2}, \tag{1b}$$

$$\partial_t(\theta h \rho_a) + \partial_i(\theta \rho_a h u^i) = \widetilde{\mathcal{S}}_a, \quad a = \overline{1,N}. \tag{1c}$$

The state variables in the model are the water depth h, the components u^i, $i = 1,2$ of the fluid velocity, and the mass densities ρ_a, $a = \overline{1,N}$ of the N size classes of the suspended sediment. The production terms are: the rate $\mathcal{W}$ of water production due to rain gain and infiltration loss, the rate $\mathcal{H}$ of momentum balance production due to vegetation resistance and soil friction, and the rate $\widetilde{\mathcal{S}}_a$ of sediment production due to soil erosion. The function θ stands for the porosity of the cover plant and z for the altitude of the initial soil surface.

2.1 Hydraulic closure relations

The rate of the momentum balance includes the vegetation resistance and fluid soil friction [1, 8, 11, 12],

$$\mathcal{H}^i = -(\alpha_v h(1-\theta) + \alpha_s \theta)|u|u^i. \tag{2}$$

The parameter α_v depends of the geometry of the plants,

$$\alpha_v = \frac{2C_D}{\pi d}, \tag{3}$$

where C_D and d are the averaged aerodynamic resistance coefficient and the averaged diameter, respectively of an individual plant. The parameter α_s is a measure of the soil surface roughness. Mainly, there are three different forms of this coefficient: Chezy, Manning and Darcy-Weisbach

$$\alpha_s = \begin{cases} \frac{g}{C_b^2}, & \text{Chezy law} \\ \frac{f}{8}, & \text{Darcy} - \text{Weisbach law} \\ \frac{g}{K^2 h^{1/3}}, & \text{Manning} - \text{Strickler law} \end{cases}, \tag{4}$$

where g is the gravitational acceleration and C_b, f are material constants. The Manning coefficient n is related to K coefficient in the Manning-Strickler law by $n = 1/K$.

2.2 Erosion model

The erosion mathematical model is adapted from the Hairsine-Rose model [5]. One considers a column of mixture composed of water and sediment. The column is raised on a 2-D ground domain Ω and partitioned into two regions: one of them (the bottom one) contains the "solid part", forms the bed of soil and is of height δ, and the other one (the top one) contains the water and the suspended sediment, and is of height h. The soil bed is composed of the original erodible soil and a part produced by the depositing of the suspended sediment. For each fraction of the sediment, the mass balance on the entire column can be read as

$$\partial t \int_{\Omega} \int_{0}^{\delta+h} \rho_a \mathrm{d}z \mathrm{d}\sigma + \int_{\partial\Omega} \int_{0}^{\delta+h} \rho_a \boldsymbol{w}_a \cdot \boldsymbol{n} \mathrm{d}z \mathrm{d}s = 0, \tag{5}$$

where δ is the height of the bed soil, h is the depth of fluid part, $\boldsymbol{w}_a$ is the velocity of the sediment fraction, and $\boldsymbol{n}$ is the exterior unitary normal to $\partial\Omega$. We assume that $\rho_a = 0$ on the domain occupied by the plants in the fluid zone, and also on the complementary of the solid matrix. It is supposed that the entire column lies on a non-erodible bed rock.

One splits the sediment column into the fluid moving part and bed soil part. If one assumes that the motion of the bed of soil along the bed rock is missing, then we can write

$$\partial t \int_{\Omega} \int_{\delta}^{\delta+h} \rho_a \mathrm{d}z \mathrm{d}\sigma + \int_{\partial\Omega} \int_{\delta}^{\delta+h} \rho_a \boldsymbol{w}_a \cdot \boldsymbol{n} \mathrm{d}z \mathrm{d}s + \partial t \int_{\Omega} \int_{0}^{\delta} \rho_a \mathrm{d}z \mathrm{d}\sigma = 0. \tag{6}$$

Following [6], after an averaging method, we obtain

$$\partial_t(\theta h \rho_a) + \partial_i(\theta \rho_a h u^i) + \partial_t \widetilde{m}_a = 0, \quad a = \overline{1, N}, \tag{7}$$

where the components of the velocity of the suspended sediment along the soil surface are assumed to be equal to the ones corresponding to the fluid velocity.

The quantity $\widetilde{m}_a$ denotes the mass of sediments corresponding to size class a per unit area in the bed soil. Its time variation is due to the soil erosion processes and depositing of the suspended sediment. These two processes take place only at the interface soil-fluid.

In the Hairsine-Rose model [5], one divides $\widetilde{m}_a$ into the original soil part m_a^s and the recirculated sediment part m_a

$$\widetilde{m}_a = m_a + m_a^s, \tag{8}$$

each of them obeying different erosion laws. If one ignores the erosion induced by rain, then the variation of the recirculated soil is determined only by the erosion due to the water flow and the depositing processes, i.e.

$$\partial_t m_a = \theta(d_a - e_a^r), \tag{9}$$

while the variation of the original bed soil is determined only by the erosion due to the water flow, i.e.

$$\partial_t m_a^s = -\theta e_a. \tag{10}$$

Consequently, the variation of the bed soil mass is given by

$$\partial_t \widetilde{m}_a = \theta(d_a - e_a^r - e_a). \tag{11}$$

Using (7) and (11), the rates of the mass sources in (1) take the form

$$\widetilde{\mathcal{S}}_a = \theta(e_a^r + e_a - d_a). \tag{12}$$

2.3 Erosion closure relations

We now adopt the closure relations proposed by [5, 10, 13]

$$\begin{aligned} d_a &= \rho_a \omega_a, \\ e_a &= p_a(1-H)\frac{F\left(\Omega - \Omega_{cr}\right)}{J}, \\ e_a^r &= H\frac{m_a}{m_t}\frac{\gamma_s}{\gamma_s - 1}\frac{F\left(\Omega - \Omega_{cr}\right)}{gh}, \end{aligned} \tag{13}$$

where

- ⋆ p_a - the proportion of the sediment in the size class a in the original soil;
- ⋆ γ_s - specific weight of sediment;
- ⋆ F - effective fraction of power stream;
- ⋆ Ω_{cr} - critical power stream;
- ⋆ $\Omega = \theta\rho_{apa}|\tau_s||u|$ - stream power, $|\tau_s| = \theta\alpha_s|u|^2$;

- $\star$ J - energy of soil particles detachment;
- $\star$ $H = \min\left\{\frac{m_t}{m_t^\star}, 1\right\}$ - the proportion of cover of the soil by recirculated sediment
    - $\circ$ $m_t = \sum_{a=1}^{N} m_a$ - total mass of sediment deposited on the soil,
    - $\circ$ $m_t^\star$ - mass required to protect the original soil from erosion.

3 Discrete CAESAR model of soil erosion on vegetated watershed

The discrete model in CAESAR [3] uses a square cell space unit to model the spatial variation. The time variation of all variables are described by a discrete dynamical system. If one denotes the vector of the state variables by $\mathfrak{U}$, the values of the state vector at the moment of time t_n and located at the unit cell $\mathcal{C}_{i,j}$ by $\mathfrak{U}^n_{i,j}$, then the time evolution is described by

$$\mathfrak{U}^{n+1}_{i,j} = \mathfrak{F}_{i,j}\left(\mathfrak{U}^n_{i,j}, t_n, \triangle t_n, \triangle A, \mathbf{\Gamma}\right), \tag{14}$$

where $\triangle t_n$ denotes the timestep from the moment of time t_n to $t_{n+1} = t_n + \triangle t_n$, $\triangle A$ the area of the cell unit, and $\mathbf{\Gamma}$ the set of all parameters in the model. $\mathfrak{F}$ represents the discrete version of all the interaction processes that govern the evolution of the state vector $\mathfrak{U}$.

The discrete model for the water flow and soil erosion implemented in CAESAR consists of

$$h^{n+1}_{i,j} = h^n_{i,j} - \lambda\left(q^n_{i+1/2,j} - q^n_{i-1/2,j} + r^n_{i,j+1/2} - r^n_{i,j-1/2}\right), \tag{15a}$$

$$c[a]^{n+1}_{i,j} = c[a]^n_{i,j} - \lambda\left(q[a]^n_{i+1/2,j} - q[a]^n_{i-1/2,j} + r[a]^n_{i,j+1/2} - r[a]^n_{i,j-1/2}\right) + \triangle t_n \mathcal{S}^n[a]_{i,j}, \tag{15b}$$

$$m[a]^{n+1}_{i,j} = \triangle t_n \theta_{i,j}(d[a]_{i,j} - e[a]^r_{i,j}), \tag{15c}$$

$$m^s[a]^{n+1}_{i,j} = -\triangle t_n \theta_{i.j} e[a]_{i,j}. \tag{15d}$$

where $\lambda = \triangle t_n/\triangle x$, $\mathcal{S}^n[a]_{i,j} = e[a]^r_{i,j} + e[a]_{i,j} - d[a]_{i,j}$, and $c[a]^n_{i,j} = \rho_a h(t^n)|_{\mathcal{C}_{i,j}}$.

The mass fluxes $q := hu^1$ and $r := hu^2$ are evaluated by using a semi-implicit scheme of time integration of a simplified form of the momentum balance equations (1b)

$$\begin{aligned} q^{n+1}_{i+1/2,j} &= \frac{q^n_{i+1/2,j} - \lambda g h^n_{i+1/2,j}\left((z+h)_{i+1,j} - (z+h)_{i,j}\right)^n}{1 + |q^n_{i+1/2,j}| \triangle t_n \mathcal{K}^n_{i+1/2,j}}, \\ r^{n+1}_{i,j+1/2} &= \frac{r^n_{i,j+1/2} - \lambda g h^n_{i,j+1/2}\left((z+h)_{i,j+1} - (z+h)_{i,j}\right)^n}{1 + |r^n_{i,j+1/2}| \triangle t_n \mathcal{K}^n_{i,j+1/2}}. \end{aligned} \tag{16}$$

The quantities $\mathcal{K}$ are given by

$$\mathcal{K} = \frac{\alpha_v h(1/\theta - 1) + \alpha_s}{h^2}.$$

The mass sediment fluxes $q[a]$ and $r[a]$ are calculated by using the water fluxes q and r by relations

$$q[a] = \frac{c[a] * q}{h}, \quad r[a] = \frac{c[a] * r}{h}.$$

The stabilty of the time integration and the requirement of the posivity of water depth and mass sediment density impose some restriction on the time step.

Remark 3.1 *In this model, we assume that θ is almost constant. The equations (15a), (15b), (15c), and (15d) are the discrete correspondence of (1a), (1c), (9), and (10), respectively.*

4 Numerical results

In order to see the response of our erosion function to the input parameters of the soil erosion, we performed a series of tests based on realistic data coming from terrain data and literature. We simulate the land erosion on Paul's Valley - a relative small watershed in the Ampoi's hydrographic basin. The parameters of the vegetative cover are hypothetical, while the ones corresponding to soil erosion are obtained from literature, [10]. The topography and hydrodynamic regimes are illustrated in Figure 1.

To test the capability of the erosion model and to highlight the influence of the vegetative cover on the soil erosion processes, we focus on two parameters: the plant cover density and the soil cohesivity. We believe that the plant roots modify the cohesivity of the soil, but we do not know in what sense. The results are presented in Figures 2 and 3.

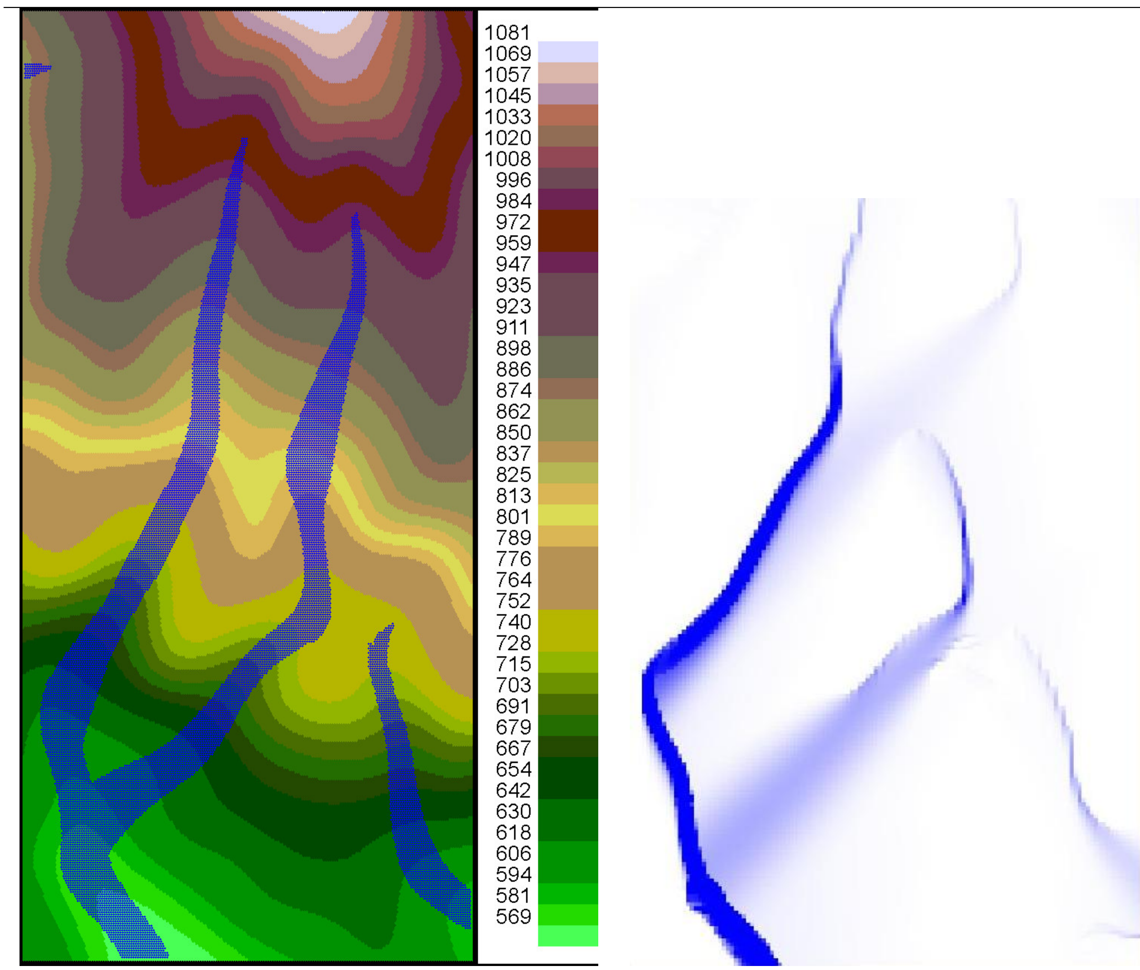
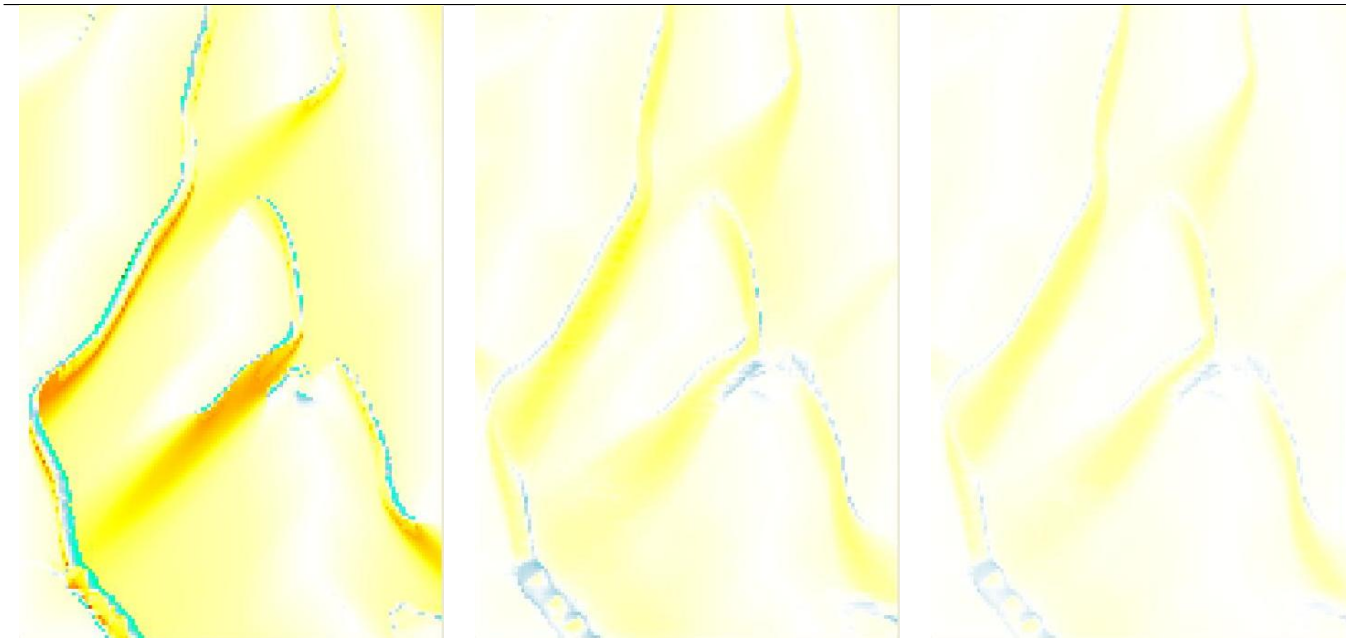
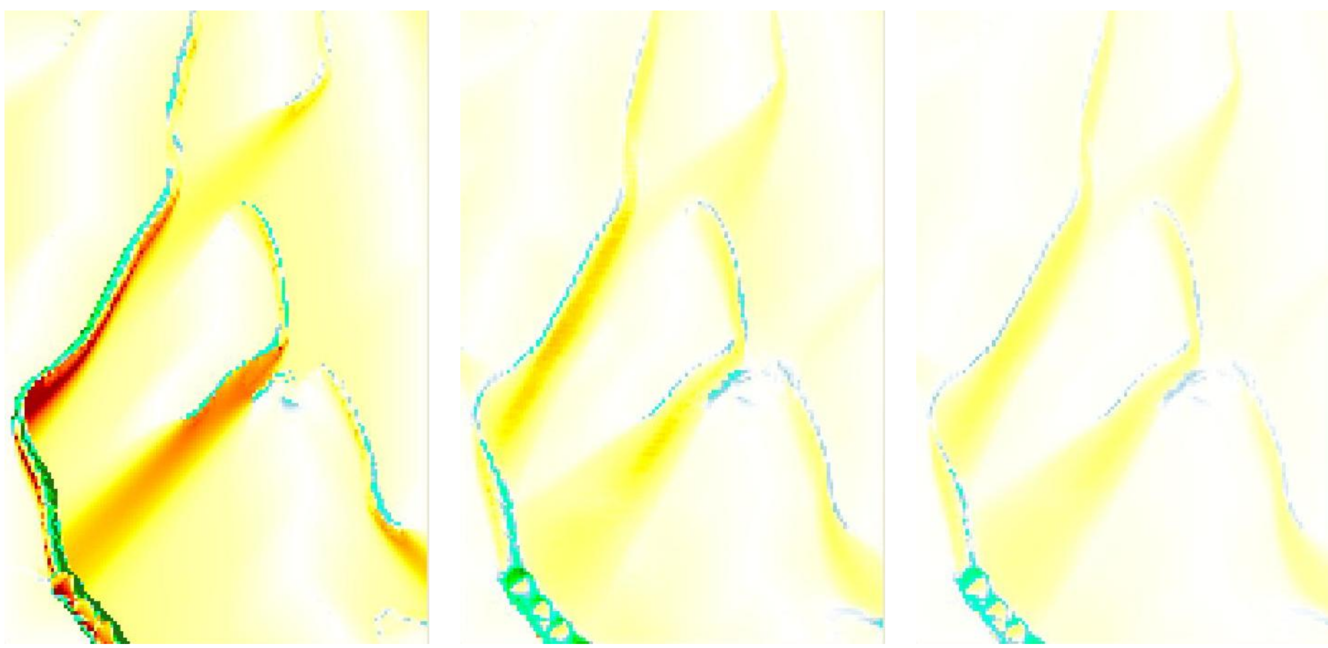


Figure 1: Water dynamics in Paul's Valley. Left picture shows the potential water accumulation zone, while the right picture illustrates the water depth distribution at a given time moment. To obtain the left picture, we use a water routing method introduced in [7], while the right picture was obtained by running CAESAR with (15).



t=20 min



t=60 min

Figure 2: Mass bed soil variation in Paul's Valley. From left to right: medium cohesive bare soil, medium cohesive vegetated soil, high cohesive vegetated soil. Blue to green gradient color: mass bed soil gain; yellow to red gradient: mass soil lost.

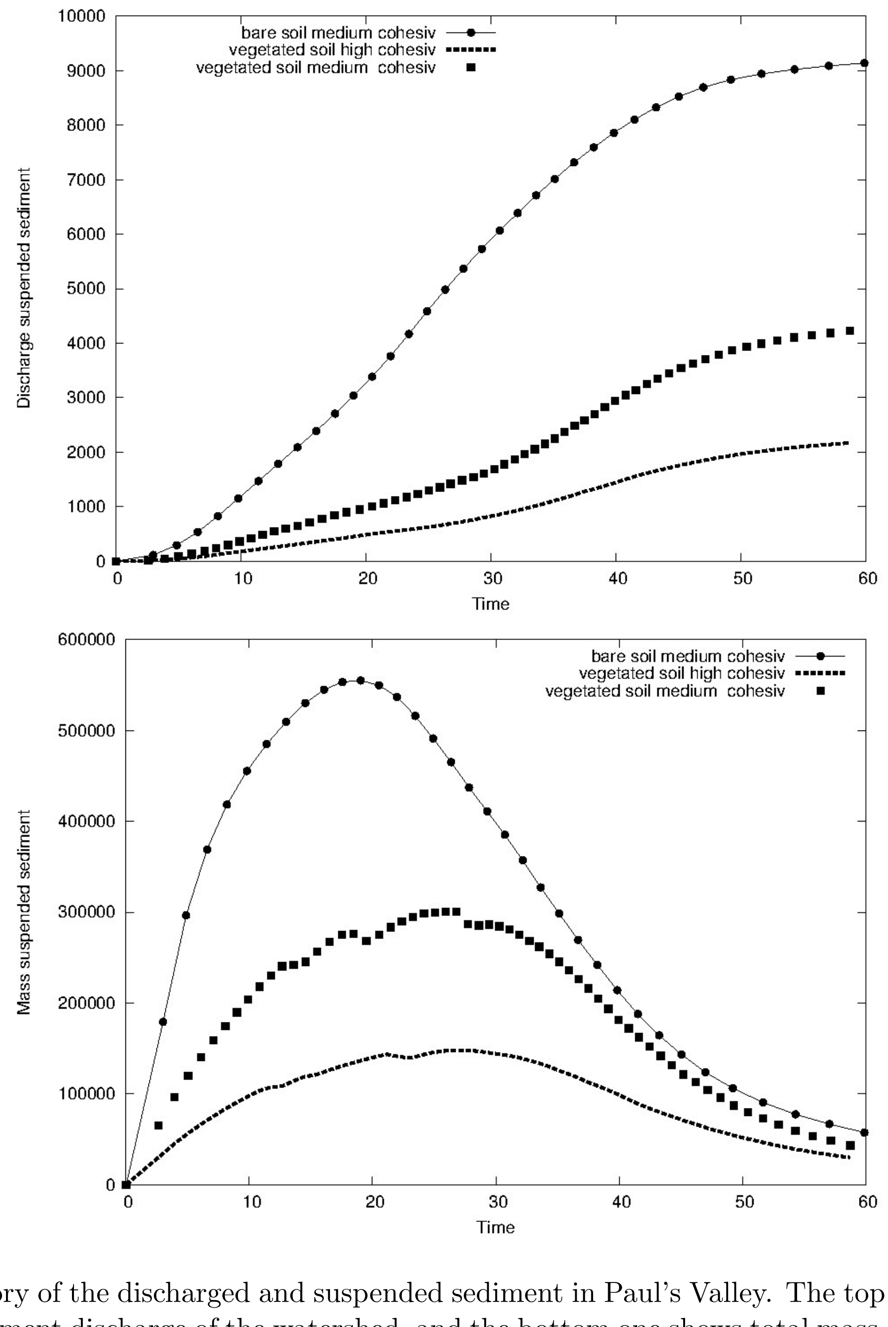


Figure 3: History of the discharged and suspended sediment in Paul's Valley. The top figure shows the cumulated sediment discharge of the watershed, and the bottom one shows total mass of the suspended sediment on the entire watershed.

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