b) $r_{1,2} = \dfrac{-p \pm \sqrt{p^2-4}}{2}$

Evidently there are complex roots for $p^2 < 4$, i.e. $0 \le p < 2$.
Hence, solutions to the differential equation osscilate when $0 \le p < 2$, but no longer oscilate for $p \ge 2$. Hence, $P_0 = 2$.

For $p = \frac{1}{2} P_0 = 1$ we have:

$$r_{1,2} = \frac{-1 \pm i\sqrt{3}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$$y(t) = e^{-\frac{1}{2}t} \cdot \left(c_1 \cos\left(\tfrac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\tfrac{\sqrt{3}}{2}t\right)\right)$$

But $0 = y(0) = c_1$

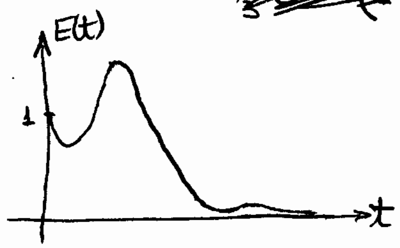
$1 = y'(0) = -\frac{1}{2}c_1 + \frac{\sqrt{3}}{2}c_2 \quad \Longrightarrow \quad c_1 = 0,\ c_2 = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Hence, $y(t) = \frac{2\sqrt{3}}{3} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$

$$E(t) = 4y^2(t) + y'(t)^2 = \frac{16}{3} e^{-t} \sin^2\left(\tfrac{\sqrt{3}}{2}t\right) + \frac{4}{3}\left(-\frac{1}{2}e^{-1/2\,t}\sin\left(\tfrac{\sqrt{3}}{2}t\right) + \frac{\sqrt{3}}{2}e^{-\frac{1}{2}t}\cos\left(\tfrac{\sqrt{3}}{2}t\right)\right)^2$$

$$= \frac{16}{3}e^{-t}\sin^2\left(\tfrac{\sqrt{3}}{2}t\right) + e^{-t}\left(\frac{1}{3}\sin^2\left(\tfrac{\sqrt{3}}{2}t\right) + \cos^2\left(\tfrac{\sqrt{3}}{2}t\right) - \frac{2}{\sqrt{3}}\sin\left(\tfrac{\sqrt{3}}{2}t\right)\cos\left(\tfrac{\sqrt{3}}{2}t\right)\right)$$

$$= \frac{1}{3}e^{-t} \cdot \left(3 + 14\sin^2\left(\tfrac{\sqrt{3}}{2}t\right) - \sqrt{3}\sin(\sqrt{3}t)\right)$$



E(t) oscillates with decaying amplitude.

c) $p = P_0 = 2 \Rightarrow r_{1,2} = -1 \Rightarrow y(t) = c_1 e^{-t} + c_2 t e^{-t}$
$0 = y(0) = c_1, \quad 1 = y'(0) = -c_1 + c_2 \quad \Rightarrow y(t) = t e^{-t}$

$p = 2P_0 = 4 \Rightarrow r_{1,2} = -2 \pm \sqrt{3} \Rightarrow y(t) = c_1 e^{(-2-\sqrt{3})t} + c_2 e^{(-2+\sqrt{3})t}$
$0 = y(0) = c_1 + c_2, \quad 1 = y'(0) = (-2-\sqrt{3})c_1 + (-2+\sqrt{3})c_2 \quad \Rightarrow$

$$\Rightarrow y(t) = \frac{1}{2\sqrt{3}}\left(-e^{(-2-\sqrt{3})t} + e^{(-2+\sqrt{3})t}\right)$$